

Package ‘jordan’

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Type Package

Title A Suite of Routines for Working with Jordan Algebras

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Description A Jordan algebra is an algebraic object originally designed to study observables in quantum mechanics. Jordan algebras are commutative but non-associative; they satisfy the Jordan identity. The package follows the ideas and notation of K. McCrimmon (2004, ISBN:0-387-95447-3) ``A Taste of Jordan Algebras". To cite in publications please use Hankin (2023) <[arXiv:2303.06062v1](https://arxiv.org/abs/2303.06062v1)>.

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Suggests knitr,rmarkdown

Depends onion (>= 1.4-0), Matrix

VignetteBuilder knitr

Imports emulator,methods,mathjaxr

URL <https://github.com/RobinHankin/jordan>

BugReports <https://github.com/RobinHankin/jordan/issues>

RdMacros mathjaxr

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Description

A Jordan algebra is an algebraic object originally designed to study observables in quantum mechanics. Jordan algebras are commutative but non-associative; they satisfy the Jordan identity. The package follows the ideas and notation of K. McCrimmon (2004, ISBN:0-387-95447-3) "A Taste of Jordan Algebras". To cite in publications please use Hankin (2023) <arXiv:2303.06062v1>.

Details

A *Jordan algebra* is a non-associative algebra over the reals with a multiplication that satisfies the following identities:

$$xy = yx$$

$$(xy)(xx) = x(y(xx))$$

(the second identity is known as the Jordan identity). In literature one usually indicates multiplication by juxtaposition but one sometimes sees $x \circ y$. Package idiom is to use an asterisk, as in $x*y$. There are five types of Jordan algebras:

1. Real symmetric matrices, class `real_symmetric_matrix`, abbreviated in the package to `rsm`
2. Complex Hermitian matrices, class `complex_herm_matrix`, abbreviated to `chm`
3. Quaternionic Hermitian matrices, class `quaternion_herm_matrix`, abbreviated to `qhm`
4. Albert algebras, the space of 3×3 octonionic matrices, class `albert`
5. Spin factors, class `spin`

(of course, the first two are special cases of the next). The `jordan` package provides functionality to manipulate jordan objects using natural `R` idiom.

Objects of all these classes are stored in dataframe (technically, a matrix) form with columns being elements of the jordan algebra.

The first four classes are matrix-based in the sense that the algebraic objects are symmetric or Hermitian matrices (the `S4` class is "jordan_matrix"). The fifth class, spin factors, is not matrix based.

One can extract the symmetric or Hermitian matrix from objects of class `jordan_matrix` using `as.list()`, which will return a list of symmetric or Hermitian matrices. A function name preceded by a "1" (for example `as.1matrix()` or `vec_to_qhm1()`) means that it deals with a single (symmetric or Hermitian) matrix.

Algebraically, the matrix form of `jordan_matrix` objects is redundant (for example, a `real_symmetric_matrix` of size $n \times n$ has only $n(n+1)/2$ independent entries, corresponding to the upper triangular elements).

Author(s)

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References

K. McCrimmon 1978. "Jordan algebras and their applications". *Bulletin of the American Mathematical Society*, Volume 84, Number 4.

Examples

```

rrsm()      # Random Real Symmetric matrices
rchm()      # Random Complex Hermitian matrices
rqhm()      # Random Quaternionic Hermitian matrices
ralbert()   # Random Albert algebra
rspin()     # Random spin factor

x <- rqhm(n=1)
y <- rqhm(n=1)
z <- rqhm(n=1)

x/1.2 + 0.3*x*y      # Arithmetic works as expected ...
x*(y*z) -(x*y)*z    # ... but '*' is not associative

## Verify the Jordan identity for type 3 algebras:

LHS <- (x*y)*(x*x)
RHS <- x*(y*(x*x))

diff <- LHS-RHS # zero to numerical precision

diff[1,drop=TRUE] # result in matrix form

```

Arith

Methods for Function Arith in package Jordan

Description

Methods for Arithmetic functions for jordan: +, -, *, /, ^

Usage

```

jordan_negative(z)
jordan_plus_jordan(e1,e2)
jordan_plus_numeric(e1,e2)
jordan_prod_numeric(e1,e2)
jordan_power_jordan(e1,e2)
albert_arith_albert(e1,e2)
albert_arith_numeric(e1,e2)
albert_inverse(e1)
albert_power_albert(...)
albert_power_numeric(e1,e2)
albert_power_single_n(e1,n)
albert_prod_albert(e1,e2)
chm_arith_chm(e1,e2)
chm_arith_numeric(e1,e2)

```

```

chm_inverse(e1)
chm_power_numeric(e1,e2)
chm_prod_chm(e1,e2)
numeric_arith_albert(e1,e2)
numeric_arith_chm(e1,e2)
numeric_arith_qhm(e1,e2)
numeric_arith_rsm(e1,e2)
qhm_arith_numeric(e1,e2)
qhm_arith_qhm(e1,e2)
qhm_inverse(x)
qhm_power_numeric(e1,e2)
qhm_prod_qhm(e1,e2)
rsm_arith_numeric(e1,e2)
rsm_arith_rsm(e1,e2)
rsm_inverse(e1)
rsm_power_numeric(e1,e2)
rsm_prod_rsm(e1,e2)
spin_plus_numeric(e1,e2)
spin_plus_spin(e1,e2)
spin_power_numeric(e1,e2)
spin_power_single_n(e1,n)
spin_power_spin(...)
spin_prod_numeric(e1,e2)
spin_prod_spin(e1,e2)
spin_inverse(...)
spin_negative(e1)
vec_albertprod_vec(x,y)
vec_chmprod_vec(x,y)
vec_qhmprod_vec(x,y)
vec_rsmprod_vec(x,y)

```

Arguments

<code>z, e1, e2</code>	Jordan objects or numeric vectors
<code>n</code>	Integer for powers
<code>...</code>	Further arguments (ignored)
<code>x, y</code>	Numeric vectors, Jordan objects in independent form

Details

The package implements the Arith group of S4 generics so that idiom like `A + B*C` works as expected with jordans.

Functions like `jordan_inverse()` and `jordan_plus_jordan()` are low-level helper functions. The only really interesting operation is multiplication; functions like `jordan_prod_jordan()`.

Names are implemented and the rules are inherited (via `onion::harmonize_oo()` and `onion::harmonize_on()`) from `rbind()`.

Value

generally return jordans

Author(s)

Robin K. S. Hankin

Examples

```
x <- rspin()
y <- rspin()
z <- rspin()

x*(y*(x*x)) - (x*y)*(x*x) # should be zero

x + y*z
```

c

Concatenation

Description

Combines its arguments to form a single jordan object.

Usage

```
## S4 method for signature 'jordan'
c(x,...)
```

Arguments

x, ... Jordan objects

Details

Returns a concatenated jordan of the same type as its arguments. Argument checking is not performed.

Value

Returns a Jordan object of the appropriate type (coercion is not performed)

Note

Names are inherited from the behaviour of `cbind()`, not `c()`.

Author(s)

Robin K. S. Hankin

Examples

```
c(rqhm(), rqhm()*10)
```

 coerce

Coercion

Description

Various coercions needed in the package

Usage

```

as.jordan(x, class)
vec_to_rsm1(x)
vec_to_chm1(x)
vec_to_qhm1(x)
vec_to_albert1(x)
rsm1_to_vec(M)
chm1_to_vec(M)
qhm1_to_vec(M)
albert1_to_vec(H)
as.real_symmetric_matrix(x, d, single=FALSE)
as.complex_herm_matrix(x, d, single=FALSE)
as.quaternion_herm_matrix(x, d, single=FALSE)
as.albert(x, single=FALSE)
numeric_to_real_symmetric_matrix(x, d)
numeric_to_complex_herm_matrix(x, d)
numeric_to_quaternion_herm_matrix(x, d)
numeric_to_albert(e1)
as.list(x, ...)
matrix1_to_jordan(x)

```

Arguments

<code>x, e1</code>	Numeric vector of independent entries
<code>M, H</code>	A matrix
<code>d</code>	Dimensionality of algebra
<code>single</code>	Boolean, indicating whether a single value is to be returned
<code>class</code>	Class of object
<code>...</code>	Further arguments, currently ignored

Details

The numeral “1” in a function name means a single element, usually a matrix.

Functions `vec_to_rsm1()` et seq convert a numeric vector to a (symmetric, complex, quaternion, octonion) matrix, that is, elements of a matrix-based Jordan algebra.

Functions `rsm1_to_vec()` convert a (symmetric, complex, quaternion, octonion) matrix to a numeric vector of independent components. The upper triangular components are used; no checking for symmetry is performed (the lower triangular components, and non-real components of the diagonal, are discarded).

Function `as.1matrix()` is used to convert a jordan object to a list of matrices. Length one jordan objects are converted to a matrix.

Functions `as.real_symmetric_matrix()`, `as.complex_herm_matrix()`, `as.quaternion_herm_matrix()` and `as.albert()` take a numeric matrix and return a (matrix-based) Jordan object.

Functions `numeric_to_real_symmetric_matrix()` have not been coded up yet.

Function `matrix1_to_jordan()` takes a matrix and returns a length-1 (matrix based) Jordan vector. It uses the class of the entries (real, complex, quaternion, octonion) to decide which type of Jordan to return.

Value

Return a coerced value.

Author(s)

Robin K. S. Hankin

Examples

```
vec_to_chm1(1:16) # Hermitian matrix

as.1matrix(rchm())

as.complex_herm_matrix(matrix(runif(75),ncol=3))

matrix1_to_jordan(cprod(matrix(rnorm(35),7,5)))
matrix1_to_jordan(matrix(c(1,1+1i,1-1i,3),2,2))
matrix1_to_jordan(0i1 + matrix(1,3,3))
```

Compare-methods

Methods for compare S4 group

Description

Methods for comparison (equal to, greater than, etc) of jordans. Only equality makes sense.

Usage

```
jordan_compare_jordan(e1,e2)
```

Arguments

e1,e2 Jordan objects

Value

Return a boolean

Examples

```
# rspin() > 0 # meaningless and returns an error
```

 extract

Extract and replace methods for jordan objects

Description

Extraction and replace methods for jordan objects should work as expected.

Replace methods can take a jordan or a numeric, but the numeric must be zero.

Value

Generally return a jordan object of the same class as the first argument

Methods

```
[ signature(x = "albert", i = "index", j = "missing", drop = "logical"): ...
[ signature(x = "complex_herm_matrix", i = "index", j = "missing", drop = "logical"): ...
[ signature(x = "jordan", i = "index", j = "ANY", drop = "ANY"): ...
[ signature(x = "jordan", i = "index", j = "missing", drop = "ANY"): ...
[ signature(x = "quaternion_herm_matrix", i = "index", j = "missing", drop = "logical"):
  ...
[ signature(x = "real_symmetric_matrix", i = "index", j = "missing", drop = "logical"):
  ...
[ signature(x = "spin", i = "index", j = "missing", drop = "ANY"): ...
[ signature(x = "spin", i = "missing", j = "index", drop = "ANY"): ...
[<- signature(x = "albert", i = "index", j = "missing", value = "albert"): ...
[<- signature(x = "complex_herm_matrix", i = "index", j = "ANY", value = "ANY"): ...
[<- signature(x = "complex_herm_matrix", i = "index", j = "missing", value = "complex_herm_matrix"):
  ...
[<- signature(x = "jordan_matrix", i = "index", j = "missing", value = "numeric"): ...
[<- signature(x = "quaternion_herm_matrix", i = "index", j = "missing", value = "quaternion_herm_matri
  ...
[<- signature(x = "real_symmetric_matrix", i = "index", j = "missing", value = "real_symmetric_matrix"):
  ...
[<- signature(x = "spin", i = "index", j = "index", value = "ANY"): ...
[<- signature(x = "spin", i = "index", j = "missing", value = "numeric"): ...
[<- signature(x = "spin", i = "index", j = "missing", value = "spin"): ...
```

Author(s)

Robin K. S. Hankin

Examples

```
showClass("index") # taken from the Matrix package

a <- rspin(7)
a[2:4] <- 0
a[5:7] <- a[1]*10
a
```

id

*Multiplicative identities***Description**

Multiplying a jordan object by the *identity* leaves it unchanged.

Usage

```
as.identity(x)
rsm_id(n,d)
chm_id(n,d)
qhm_id(n,d)
albert_id(n)
spin_id(n=3,d=5)
```

Arguments

n	Length of vector to be created
d	Dimensionality
x	In function <code>as.identity()</code> , a jordan object. Return value will be a jordan object of the same dimensionality but entries equal to the identity

Details

The identity object in the matrix-based classes (`jordan_matrix`) is simply the identity matrix. Function `as.identity()` takes an object of any of the five types (`rsm`, `chm`, `qhm`, `spin`, or `albert`) and returns a vector of the same length and type, but comprising identity elements.

Class `spin` has identity $(1, \mathbf{0})$.

Value

A jordan object is returned.

Author(s)

Robin K. S. Hankin

Examples

```
x <- as.albert(matrix(sample(1:99,81,replace=TRUE),nrow=27))
I <- as.identity(x)
x == x*I # should be TRUE

rsm_id(6,3)
```

jordan

Create jordan objects

Description

Creation methods for jordan objects

Arguments

M	A matrix with columns representing independent entries in a matrix-based Jordan algebra
a, V	Scalar and vector components of a spin factor

Details

The functions documented here are the creation methods for the five types of jordan algebra.

- quaternion_herm_matrix()
- complex_herm_matrix()
- real_symmetric_matrix()
- albert()
- spin()

(to generate quick “get you going” Jordan algebra objects, use the `rasm()` family of functions, documented at `random.Rd`).

Value

Return jordan objects or Boolean as appropriate

Author(s)

Robin K. S. Hankin

See Also

[random](#)

Examples

```

A <- real_symmetric_matrix(1:10) # vector of length 1
as.1matrix(A)                    # in matrix form

complex_herm_matrix(cbind(1:25,2:26))
quaternion_herm_matrix(1:15)

albert(1:27)
spin(-6,cbind(1:12,12:1))

x <- rrsn() ; y <- rrsn() ; z <- rrsn() # also works with the other Jordans

x*(y*z) - (x*y)*z          # Jordan algebra is not associative...
(x*y)*(x*x) - x*(y*(x*x)) # but satisfies the Jordan identity

```

jordan-class

Classes in the "jordan" package

Description

Various classes in the **jordan** package.

Author(s)

Robin K. S. Hankin

References

K. McCrimmon 1978. "Jordan algebras and their applications". *Bulletin of the American Mathematical Society*, Volume 84, Number 4.

Examples

```
showClass("jordan")
```

misc

Miscellaneous Jordan functionality

Description

Miscellaneous Jordan functionality that should be documented somewhere

Usage

```

harmonize_spin_numeric(e1,e2)
harmonize_spin_spin(e1,e2)

```

Arguments

e1, e2 Objects to harmonize

Details

Miscellaneous low-level helper functions.

The harmonize functions `harmonize_spin_numeric()` and `harmonize_spin_spin()` work for spin objects for the matrix-based classes `emulator::harmonize_foo()` is used.

Value

These are mostly low-level helper functions; they not particularly user-friendly. They generally return either numeric or Jordan objects.

Author(s)

Robin K. S. Hankin

r_to_n *Sizes of Matrix-based Jordan algebras*

Description

Given the number of rows in a (matrix-based) Jordan object, return the size of the underlying associative matrix algebra

Usage

```
r_to_n_rsm(r)
r_to_n_chm(r)
r_to_n_qhm(r)
r_to_n_albert(r=27)
n_to_r_rsm(n)
n_to_r_chm(n)
n_to_r_qhm(n)
n_to_r_albert(n=3)
```

Arguments

n Integer, underlying associative algebra being matrices of size $n \times n$
r Integer, number of rows of independent representation of a matrix-based jordan object

Details

These functions are here for consistency, and the `albert` ones for completeness.

For the record, they are:

- Real symmetric matrices, `rsm`, $r = n(n + 1)/2$, $n = (\sqrt{1 + 4r} - 1)/2$
- Complex Hermitian matrices, `chm`, $r = n^2$, $n = \sqrt{r}$
- Quaternion Hermitian matrices, `qhm`, $r = n(2n - 1)$, $n = (1 + \sqrt{1 + 8r})/4$
- Albert algebras, $r = 27$, $n = 3$

Value

Return non-negative integers

Note

I have not been entirely consistent in my use of these functions.

Author(s)

Robin K. S. Hankin

Examples

```
r_to_n_qhm(nrow(rqhm()))
```

random

Random Jordan objects

Description

Random jordan objects with specified properties

Usage

```
ralbert(n=3)
rrsm(n=3, d=5)
rchm(n=3, d=5)
rqhm(n=3, d=5)
rspin(n=3, d=5)
```

Arguments

n	Length of random object returned
d	Dimensionality of random object returned

Details

These functions give a quick “get you going” random Jordan object to play with.

Value

Return a jordan object

Author(s)

Robin K. S. Hankin

Examples

```
rrsm()
ralbert()
rspin()
```

show

Print methods

Description

Show methods, to display objects at the prompt

Usage

```
albert_show(x)
spin_show(x)
jordan_matrix_show(x)
description(x, plural=FALSE)
```

Arguments

x	Jordan object
plural	Boolean, indicating whether plural form is to be given

Details

The special algebras use a bespoke show method, `jordan_matrix_show()` or `spin_show()`. If the number of elements is small, they display a concise representation and modify the row and column names of the underlying matrix slightly; spin factors are displayed with the scalar component offset from the vector component.

Print methods for special algebras are sensitive to the value of option `head_and_tail`, a two-element integer vector indicating the number of start lines and end lines to print.

Function `description()` gives a natural-language description of its argument, used in the print method.

Value

Returns the argument

Author(s)

Robin K. S. Hankin

Examples

```
rspin()
rqhm()
rchm()
```

valid	<i>Validity methods</i>
-------	-------------------------

Description

Validity methods, to check that objects are well-formed

Usage

```
valid_rsm(object)
valid_chm(object)
valid_qhm(object)
valid_albert(object)
is_ok_rsm(r)
is_ok_chm(r)
is_ok_qhm(r)
is_ok_albert(r)
is_ok_rsm(r)
```

Arguments

object	Putative jordan object
r	Integer, number of rows in putative jordan object

Details

Validity methods. The `validity_foo()` functions test for an object to be the right type, and the `is_ok_foo()` functions test the number of rows being appropriate for a jordan object of some type; these functions return an error if not appropriate, or, for `jordan_matrix` objects, the size of the matrix worked with.

Value

Return a Boolean

Author(s)

Robin K. S. Hankin

Examples

```
is_ok_qhm(45) # 5x5 Hermitian quaternionic matrices
#is_ok_qhm(46) # FALSE
```

`zero`*The zero Jordan object*

Description

Package idiom for the zero Jordan object, and testing

Usage

```
is.zero(x)
is_zero_jordan(e1, e2=0)
```

Arguments

<code>x, e1</code>	Jordan object to test for zeroness
<code>e2</code>	Dummy numeric object to make the <code>Arith</code> method work

Details

One often wants to test a jordan object for being zero, and natural idiom would be `rchm()==0`. The helper function is `is_zero_jordan()`, and the generic is `is.zero()`.

Value

Returns a Boolean

Author(s)

Robin K. S. Hankin

Examples

```
rism()*0 == 0
```


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