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## Transcriber's notes

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# AMERICAN SOCIETY OF CIVIL ENGINEERS 

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## TRANSACTIONS

Paper No. 1167
EXPANSION OF PIPES.
By Ralph C. Taggart, Assoc. M. Am. Soc. C. E.
With Discussion by Messrs. William D. Ennis, William Kent, and Ralph C. Taggart.

In the arrangement of steam piping (or other piping, the temperature of which is subject to considerable change), proper allowance must be made for expansion. Where the change in temperature, and hence the amount of expansion, is small, the stress may come well within the elastic limit of the metal. In such cases, of course, special arrangements to care for the expansion may not be required.

The calculation to determine the allowable stress in pipe may be readily made. In the case of ordinary iron pipe, we have the following:

The modulus of elasticity of wrought iron, or the stress divided by the strain, equals 29000000 .

The coefficient of expansion of wrought iron, or the increase in length per degree Fahrenheit per unit length, is 0.00000673 .

The stress per degree Fahrenheit, therefore, would be 29000000 times 0.00000673 , which is equal to 195.2 lb . per sq. in. per degree Fahrenheit difference in temperature. For a change in temperature of $100^{\circ}$ Fahr., the stress would become 19520 lb . per sq. in., which is more than the safe working stress in the iron, especially when it is considered that the stress would be largely increased at the various screw joints, where the thickness of the pipe is reduced by the depth of the thread.

For ordinary steam apparatus the change in temperature is at least $150^{\circ}$ Fahr., so that it becomes impossible for the elasticity of the metal to care for the expansion, even if the piping is very securely tied down. For, when the elastic limit of the metal is reached, a permanent set
will result, and if this change in the form of the piping is repeated, a rupture may be expected.

In steam piping, expansion is cared for by two general methods: First, by the use of so-called expansion joints; and second, by the arrangement of the piping, so that the expansion is cared for by the spring of the piping itself.

In apparatus where the straight runs of pipe have not been too long, the second method has been used almost exclusively, although the allowance for expansion has usually been one of judgment or guesswork, and not a matter of calculation.

Where the expansion has been considerable at any one place, it has been common practice for the designing engineer to resort to the use of so-called expansion joints. There are numerous types of these joints, and although many of them have merit, the writer believes that, for many purposes, there are objections to all types. One of the bestknown types is made with one metal cylinder sliding or slipping within another. There is, ordinarily, a packed gland or stuffing-box to prevent leakage. An expansion joint of this type should always be anchored, and the pipe which moves within it should also be anchored at a point some distance from it-the distance being determined by the amount of expansion which this particular joint should care for. If the pipe and expansion joint are not thus anchored, the movement of the pipe and the thrust of the steam pressure may carry the inner cylinder of the expansion joint entirely away from the outer cylinder in which it moves. This type requires more or less packing, and although this may not be an important item if only a few expansion joints are used, and if they can be gotten at readily, nevertheless it becomes very important where an engineer has to look after a number of these joints, or where they cannot be reached with the greatest ease.

In a second type of expansion joint, a circular metal disk is fixed at its outer circumference and attached to the expanding pipe near its center. The expansion is taken care of by the spring in the metal disk, and, for this reason, the amount is usually quite small.

A third type of expansion joint is made up of what may be described as a copper pipe with deep corrugations, reinforced with steel rings. Under certain conditions this joint has been very unsatisfactory. Where it has been subjected to varying temperatures, as, for example, in a heating apparatus where the steam pressure is more or less inter-
mittent, the movement in the copper has resulted in breaking at the corrugations. It is claimed, however, that some good results have been obtained where the steam pressure was not very high, and where the pressure and temperature have been very constant.

Some authorities have suggested the use of fittings arranged so that the expansion will be cared for by the twisting of the pipe within the thread of the fitting. This has been done in some cases in low-pressure work, but a little thought or experience will convince one that it is not a method to be relied on, for as soon as the slightest actual twist occurs within the fitting, the pipe becomes loose, and the joint formed by any white lead or varnish is broken. This destroys the effect of the white lead or varnish, and the difficulty of making an ordinary pipe joint tight without some such cement is well known. In many cases, where it is thought that the expansion is cared for by a twisting in a fitting, a careful examination will show that it is really cared for entirely by the spring of the pipe, and it may be set down as a safe rule that, if there is actually a twist in the pipe-thread, due to expansion, there will almost surely be a leak, even where the pressures are low.

It may be interesting, here, to mention what is known as water packing. A so-called steam-tight joint is sometimes made where one piece of metal slips within another, a few circular rings or grooves being cut in one of the cylinders. The fit, of course, must be very good, and the idea is that the condensed steam in the rings or grooves forms a sort of packing. This arrangement is used with engine indicators and with some reducing-pressure valves of the piston type, where a steamtight joint is desired and where one cylinder must slip within the other. The success of the joint depends on two things: First, and principally, on very accurate workmanship; and second, on the fact that if a very little steam passes through the joint, any part of it which is condensed will evaporate immediately and pass away unnoticed. This is very soon proven, if the discharge from a reducing-pressure valve of this type is closed, and the line leading to it fills with water, when it will be seen that water is leaking from the joint. This is one reason for the old saying that it is easier to make a joint steam-tight than water-tight.

The most common way in which expansion is cared for in steam piping is by the spring or bending of the pipes, where a change in direction occurs, and, on the whole, this method is the most satisfactory. The allowance to be made for expansion, or the length of the spring
pieces, however, is usually guessed at, or is determined by experience, rather than by accurate calculation.

Some years ago, the writer made calculations of the lengths of spring pieces for a large underground installation, and, from these calculations, he made a number of diagrams, which he has used to a considerable extent since that time. More recently, however, the original calculations have been somewhat extended, and this paper contains the resulting diagrams and curves, both new and old, together with a short explanation of their derivation and use. It is believed that they will be of value to designing engineers and others.

Fig. 1 represents two lengths of pipe, $l_{1}$ and $l_{2}$, connected by a $90^{\circ}$ elbow. The lengths, $l_{1}$ and $l_{2}$, are supposed to represent the distances from the elbow to the points at which the pipe is held


Fig. 1. in line, or at which the pipe, if horizontal or vertical before expansion, must remain horizontal or vertical after expansion. It will be assumed that the principal expansion acts in a direction at right angles to $l_{1}$ and that the secondary or smaller relative expansion, if any, acts at right angles to $l_{2}$. Consider, first, a condition in which the secondary expansion is zero. The expansion is then at right angles to $l_{1}$ and while the spring in the length of pipe, $l_{1}$ must care largely for the expansion, the length, $l_{2}$, is also a determining factor. If $l_{2}$ becomes zero, or if the pipe at both ends of the length, $l_{1}$ is held horizontally, it is easy to determine the length of $l_{1}$ required for any given expansion, when the size of the pipe is known.

Under these conditions the formula may be worked out, and will be found to be as follows:

$$
\begin{equation*}
l_{1}{ }^{2}=\frac{87000000 D r}{f} \tag{1}
\end{equation*}
$$

where the modulus of elasticity is taken as that of wrought iron or steel,
viz., 29000000.
Where $\quad l_{1}=$ the length of pipe under strain, in inches;
$r=$ the expansion, in inches, at right angles to the length of pipe, $l_{1}$;
$D=$ the outside diameter of pipe, in inches; and,
$f=$ the maximum fiber stress, in pounds per square inch.

This does not allow for the weakening of the pipe at the fitting.
In the case under consideration, the maximum strain occurs at both ends of the length, $l_{1}$ of the pipe, and therefore the lessening in strength at the elbow or fitting should be considered, and allowance made therefor. For example, if the pipe is weaker than the fitting at this point, and if the pipe-threads cut into or reduce the effective section of the pipe to two-thirds of its normal section, the strain calculated should be reduced to two-thirds. The same result is accomplished by calculating for the usual strain, with an increase in expansion to one and one-half, for the reason that the maximum fiber stress varies directly as the expansion.

In this connection it is useful to note the following relations which hold true in the equation, $l_{1}^{2}=\frac{87000000 \mathrm{Dr}}{f}$, and also, in general, in other similar equations which will be developed later.

Other quantities remaining constant ( $f$ varies directly as $r$ ), or for a fixed length and size of pipe, the maximum fiber stress varies directly as the amount of expansion.

Other quantities remaining constant ( $f$ varies directly as $D$ ), or for a fixed length and expansion of pipe, the maximum fiber stress varies directly as the diameter of the pipe.

Other quantities remaining constant ( $r$ varies inversely as $D$ ), or for a fixed length and maximum fiber stress, the expansion varies inversely as the outside pipe diameter.

Other quantities remaining constant ( $f$ varies inversely as $l^{2}$ ), or for a fixed pipe diameter and expansion, the maximum fiber stress varies inversely as the square of the length.

Other quantities remaining constant ( $r$ varies directly as $l^{2}$ ), or for a fixed pipe diameter and maximum fiber stress, the expansion varies directly as the square of the length.

If the length, $l_{2}$, is to be considered, as well as the length, $l_{1}$, the solution of the problem becomes much more complex, but it can be worked out in a manner similar to the solution of the problem of a continuous girder. The solution is given in the following discussion.

Consider first a pipe with two lengths, $l_{1}$ and $l_{2}$, at right angles, joined together with an elbow at $a$. The lengths, $a c$ and $a d$, or $l_{1}$ and $l_{2}$, are supposed to represent the distances from the elbow to the points at which the pipes pass through walls or are otherwise held at all times in line. Consider now that an expansion occurs in the pipes, with a slight


Fig. 2.
movement, if necessary, through the two restraining walls, so that the pipes assume the new position, $c-b-d$. We will assume the principal expansion to be at right angles to $l_{1}$, or in the direction, $l_{2}$, and the secondary or smaller relative expansion will be at right angles to $l_{2}$, or in the direction, $l_{1}$. The secondary expansion need not necessarily be less in quantity than the principal expansion, but it is usually less than $\left(\frac{l_{2}}{l_{1}}\right)^{2}$ times the principal expansion. The reason for this will become more apparent as the discussion proceeds, but, of course, it is due to the fact that the expansion largely cared for by $l_{1}$ is that at right angles to $l_{1}$ and, similarly, the expansion largely cared for by $l_{2}$ is that at right angles to $l_{2}$, and also because the expansion possible varies as the square of the length of pipe under strain.

Now consider the length, $b-d$, swung through $90^{\circ}$, with the point, $b$, as a center. It will assume the new position, $b-e$. This will change in no way the conditions of stress, if the elbow is considered as a part of the pipe, and it will give an arrangement to which the formula for continuous girders can easily be applied. The walls at $c$ and $e$ are points of support, and the pipes may be considered as horizontal at these points.

The unknown load, $P$, will act at $b$. The difference in elevation between $c$ and $b$, will be called $r$, and the difference in elevation between $b$ and $e$, will be called $s$. The principal expansion is then equal to $r$, and the secondary expansion to $s$. The total horizontal length between $c$ and $e$ will also be considered as $l_{1}+l_{2}$. It is, in fact, practically $l_{1}+l_{2}+s$, but since $s$ is ordinarily a negligible quantity, as compared with $l_{1}$ and $l_{2}$, it will be neglected in this connection, although it may be considered in any special case, if desired.


Fig. 3.

The three-moment equation for continuous girders, with not more than one concentrated load on each span, may be written:

$$
\begin{align*}
\frac{M_{n}}{3}\left(l_{n}+l_{n-1}\right) & +M_{n-1} \frac{l_{n-1}}{6}+M_{n+1} \frac{l_{n}}{6}+\frac{P_{n} a_{n}}{6 l_{n}}\left(l_{n}^{2}-a_{n}{ }^{2}\right) \\
& +\frac{P_{n-1} a_{n-1}}{6 l_{n-1}}\left(l_{n-1}^{2}-a_{n-1}^{2}\right) \\
= & E I\left(\frac{Y_{n+1}-Y_{n}}{l_{n}}+\frac{Y_{n-1}-Y_{n}}{l_{n-1}}\right)  \tag{2}\\
M_{n-1}= & \text { moment at support, } n-1 ; \\
M_{n}= & \text { moment at support, } n ; \\
M_{n+1}= & \text { moment at support, } n+1 ;
\end{align*}
$$

$$
\begin{aligned}
P_{n-1} & =\text { concentrated load on span, } l_{n-1} ; \\
P_{n} & =\text { concentrated load on span, } l_{n} ; \\
l_{n-1} & =\text { distance between supports, } n-1 \text { and } n ; \\
l_{n} & =\text { distance between supports, } n \text { and } n+1 ; \\
X_{n} & =\text { distance from origin to point of application of } P_{n} ; \\
X_{n-1} & =\text { distance from origin to point of application of } P_{n-1} ; \\
a_{n} & =l_{n}-X_{n} ; \\
a_{n-1} & =l_{n-1}-X_{n-1} ; \\
E & =\text { modulus of elasticity; } \\
I & =\text { moment of inertia of section; } \\
Y_{n+1}, Y_{n}, \text { and } Y_{n-1} & =\text { ordinates of points of support, } n+1, n, \text { and } n-1 .
\end{aligned}
$$

In this case first assume, as in Fig. 4, that $n$ is at $c$ and $n+1$ at $e$, noting that the origin is at $n$.
$l_{n-1}$ and $l_{n+1}$ will then equal zero.
Let $h$ equal the difference in elevation between $c$ and $e$ or $(r-s)$.
In all cases the moment at $c$ will be called $M_{1}$, and the moment at $e, M_{2}$. Then, from Equation 2:

$$
\begin{equation*}
\frac{M_{1} l_{n}}{3}+\frac{M_{2} l_{n}}{6}+\frac{P_{n} a_{n}}{6 l_{n}}\left(l_{n}^{2}-a_{n}^{2}\right)=-E I \frac{h}{l_{n}} \tag{3}
\end{equation*}
$$



Fig. 4.


Fig. 5.

If we now assume, as in Fig. 5, that $n-1$ is at $c$ and $n$ at $e$, noting again that the origin is at $n$, we will have, from Equation 2:

$$
\begin{equation*}
\frac{M_{2} l_{n-1}}{3}+\frac{M_{1} l_{n-1}}{6}+\frac{P_{n-1} a_{n-1}}{6 l_{n-1}}\left(l_{n-1}^{2}-a_{n-1}^{2}\right)=E I \frac{h}{l_{n-1}} . \tag{4}
\end{equation*}
$$

Substituting, in Equation 4, the values used in Equation 3 for $l_{n-1}$; $P_{n-1} ; a_{n-1} ;$ viz., $l_{n} ; P_{n} ; l_{n}-a_{n}$, we have:

$$
\begin{equation*}
\frac{M_{2} l_{n}}{3}+\frac{M_{1} l_{n}}{6}+\frac{P_{n} a_{n}}{6 l_{n}}\left(2 l_{n}^{2}-3 a_{n} l_{n}+a_{n}^{2}\right)=E I \frac{h}{l_{n}} \tag{5}
\end{equation*}
$$

If we make $L$ equal to $l_{n} ; P$ equal to $P_{n}$; and $A$ equal to $a_{n}$, Equations 3 and 5 will then reduce to

$$
\begin{align*}
& 2 M_{1} L+M_{2} L+\frac{P A}{L}\left(L^{2}-A^{2}\right)=-6 E I \frac{h}{L}  \tag{6}\\
& M_{1} L+2 M_{2} L+\frac{P A}{L}\left(2 L^{2}-3 A L+A^{2}\right)=6 E I \frac{h}{L} . \tag{7}
\end{align*}
$$

Whence, by multiplying Equation 6 by 2 and subtracting Equation 7:

$$
3 M_{1} L+\frac{2 P A}{L}\left(L^{2}-A^{2}\right)-\frac{P A}{L}\left(2 L^{2}-3 A L+A^{2}\right)=-18 E I \frac{h}{L}
$$

or

$$
\begin{equation*}
M_{1}=-6 E I \frac{h}{L^{2}}-\frac{P A^{2}(L-A)}{L^{2}} \tag{8}
\end{equation*}
$$

In a similar manner, by multiplying Equation 7 by 2 and subtracting Equation 6:

$$
3 M_{2} L+\frac{2 P A}{L}\left(2 L^{2}-3 A L+A^{2}\right)-\frac{P A}{L}\left(L^{2}-A^{2}\right)=18 E I \frac{h}{L}
$$

or

$$
\begin{equation*}
M_{2}=6 E I \frac{h}{L^{2}}-\frac{P A}{L^{2}}(L-A)^{2} \tag{9}
\end{equation*}
$$

Then Equations 8 and 9 give the moments at the two points of support.

The shear just at the right of the support at $c$, may be expressed as follows:

$$
\begin{equation*}
F=\frac{M_{2}-M_{1}+P A}{L} . \tag{10}
\end{equation*}
$$

Substituting the values of $M_{1}$ and $M_{2}$, as shown in Equations 8 and 9,

$$
F=\frac{1}{L}\left(6 E I \frac{h}{L^{2}}-\frac{P A}{L^{2}}(L-A)^{2}+6 E I \frac{h}{L^{2}}+\frac{P A^{2}(L-A)}{L^{2}}+P A\right)
$$

or

$$
\begin{equation*}
F=12 E I \frac{h}{L^{3}}+\frac{P A^{2}}{L^{3}}(3 L-2 A) \tag{11}
\end{equation*}
$$

The three-moment Equation 2 is derived fundamentally from two equations (Figs. 3 and 4), namely,

$$
\begin{array}{ll}
\text { When } & X<X_{n} ; \quad M=M_{n}+F_{n} X \\
\text { When } & X>X_{n} ; \quad M=M_{n}+F_{n} X-P_{n}\left(X-X_{n}\right) \tag{13}
\end{array}
$$

When $M$ equals the moment at any point, $F_{n}$ equals the shear at the right of the support, $n$.

Since

$$
\frac{d^{2} y}{d X^{2}}=\frac{M}{E I} ; \quad M=\frac{d^{2} y}{d X^{2}} E I
$$

Substituting this value in Equations 12 and 13, integrating and determining constants, we will have:

Where $\alpha_{1}$ equals slope distance, $X$, to right of origin, $\alpha_{n}$ equals value of $\alpha_{1}$ when $X$ equals 0 .

$$
\begin{align*}
& \text { For } \quad \begin{aligned}
X<X_{n} ; \quad E I \frac{d y}{d X}= & E I \tan \alpha_{n}+M_{n} X+F_{n} \frac{X^{2}}{2} \\
X>X_{n} ; \quad E I \frac{d y}{d X}= & E I \tan \alpha_{n}+M_{n} X \\
& +F_{n} \frac{X^{2}}{2}-\frac{P_{n}\left(X-X_{n}\right)^{2}}{2}
\end{aligned} \tag{14}
\end{align*}
$$

Integrating again, and determining constants, we have:

$$
\begin{align*}
& \text { For } \quad X<X_{n} ; \quad E I y=E I X \tan \alpha_{n}+M_{n} \frac{X^{2}}{2}+F_{n} \frac{X^{3}}{6}  \tag{16}\\
& \qquad \begin{aligned}
& X>X_{n} ; \quad E I y=E I X \tan . \alpha_{n}+M_{n} \frac{X^{2}}{2} \\
&+F_{n} \frac{X^{3}}{6}-\frac{P_{n}\left(X-X_{n}\right)^{3}}{6}
\end{aligned}
\end{align*}
$$

If we now give $X$ the value $X_{n}$, and $Y$ the value $-r$, in either Equations 16 or 17 (viz., substitute the value of $X$ and $Y$ at the point of application of $P$ ), we will have:

With tan. $\alpha_{n}=0$

$$
\begin{equation*}
-E I r=M_{n} \frac{X_{n}^{2}}{2}+F_{n} \frac{X_{n}^{3}}{6} \tag{18}
\end{equation*}
$$

In this equation make $M_{n}$ equal to the value of $M_{1}$ in Equation 8; make $F_{n}$ equal to the value of $F$ in Equation 11, and substitute for $X_{n}$ the value $L-A$. We will then have:

$$
\begin{align*}
-E I r=\left(-6 E I \frac{h}{L^{2}}\right. & \left.-\frac{P A^{2}(L-A)}{L^{2}}\right) \frac{(L-A)^{2}}{2} \\
& +\left[12 E I \frac{h}{L^{3}}+\frac{P A^{2}}{L^{3}}(3 L-2 A)\right] \frac{(L-A)^{3}}{6} \tag{19}
\end{align*}
$$

or simplifying,

$$
\begin{equation*}
E I r=\frac{P A^{3}}{3 L^{3}}(L-A)^{3}+\frac{E I h}{L^{3}}\left(L^{3}-3 A^{2} L+2 A^{3}\right) . \tag{20}
\end{equation*}
$$

This gives the following value for $P$ :

$$
\begin{equation*}
P=\frac{3 L^{3} E I r-3 E I h\left(L^{3}-3 A^{2} L+2 A^{3}\right)}{A^{3}(L-A)^{3}} . \tag{21}
\end{equation*}
$$

The bending moment will be a maximum at the wall, $c$, where the bending moment is $M_{1}$, therefore,

$$
\begin{equation*}
\frac{f I}{\frac{1}{2} D}=M_{1} . \tag{22}
\end{equation*}
$$

Substituting the value of $M_{1}$ from Equation 8:

$$
\begin{equation*}
\frac{f I}{\frac{1}{2} D}=-6 E I \frac{h}{L^{2}}-\frac{P A^{2}(L-A)}{L^{2}} . \tag{23}
\end{equation*}
$$

From which:

$$
\begin{equation*}
P=\frac{-\frac{L^{2} f I}{\frac{1}{2} D}-6 E I h}{A^{2}(L-A)} \tag{24}
\end{equation*}
$$

Equating the values of $P$, as found in Equations 21 and 24, we have:

$$
\begin{equation*}
\frac{3 L^{3} E \operatorname{Ir}-3 E \operatorname{Ih}\left(L^{3}-3 A^{2} L+2 A^{3}\right)}{A^{3}(L-A)^{3}}=\frac{-\frac{L^{2} f l}{\frac{1}{2} D}-6 E I h}{A^{2}(L-A)} . \tag{25}
\end{equation*}
$$

Simplifying

$$
\begin{align*}
& -3 E r=\frac{2 f A}{D L}(L-A)^{2}-\frac{3 E h}{L^{2}}(L-A)^{2}  \tag{26}\\
& r=-\frac{2 f A(L-A)^{2}}{3 D E L}+\frac{h}{L^{2}}(L-A)^{2} . \tag{27}
\end{align*}
$$

If we make $k L=X_{n}$ and $L-k L=A$, then $A=(1-k) L$ and, substituting in Equation 27, we have:

$$
\begin{equation*}
r=-\frac{2 f(1-k) L^{2} k^{2}}{3 D E}+h k^{2} . \tag{28}
\end{equation*}
$$

If we substitute the value $(r-s)$ for $h$, we will have:

$$
\begin{equation*}
r=\frac{-2 f(1-k) L^{2} k^{2}}{3 D E}+(r-s) k^{2} \tag{29}
\end{equation*}
$$

or

$$
\begin{equation*}
r\left(1-k^{2}\right)+s k^{2}=-\frac{2 f(1-k) L^{2} k^{2}}{3 D E} . \tag{3}
\end{equation*}
$$

Or since $L k=l_{1}$

$$
\begin{equation*}
r(1+k)+s \frac{k^{2}}{1-k}=\frac{-2 f l_{1}^{2}{ }^{2}}{3 D E} . \tag{31}
\end{equation*}
$$

Where $r=$ principal expansion, in inches;
$s=$ secondary expansion, in inches;
$k=\frac{l_{1}}{l_{1}+l_{2}} \quad\left(\right.$ or $\left.\frac{l_{1}}{L}\right) ;$
$L=l_{1}+l_{2}$ (all in inches);
$f=$ maximum fiber stress (pounds per square inch);
$D=$ outside diameter of pipe, in inches; and,
$E=$ modulus of elasticity.
If $s$ equals 0 , that is, if the secondary expansion is zero, then:

$$
\begin{equation*}
r(1+k)=-\frac{2 l_{1}{ }^{2} f}{3 D E} . \tag{32}
\end{equation*}
$$

$$
\begin{align*}
& \text { If } k=1 ; l_{2}=0 \text {; and } r=-\frac{l_{1}^{2} f}{3 D E} \text {, or } \\
& \qquad l_{1}^{2}=\frac{-3 D E r}{f}=\frac{-87000000 D r}{f} \tag{33}
\end{align*}
$$

which is the same as Equation 1.
This is the condition of a beam, both ends of which are held horizontal while one end is forced to a lower level than the other.

If, on the other hand, $s$ equals zero and $k$ approaches zero (which shows that $l_{2}$ becomes very long as compared with $l_{1}$ ) then $r$ approaches a value $-\frac{2 l_{1}^{2} f}{3 D E}$, and would become equal to it in the limit. This is, of course, the condition of a beam fixed at one end and free at the other. In this case the length, $l_{2}$, would act as if it (the pipe) were cut off at the elbow. It should be noted that the value of $r$, when $k$ equals zero, or $l_{2}$ equals infinity, is twice that when $k$ equals 1 , or $l_{2}$ equals zero.

From Equation 31 certain curves have been worked out, which can be used when it is not desired to solve the equation for each independent case. The method of using these curves will be shown for a number of cases.

The curves, Figs. 6 to 22, are calculated (for wrought-iron or steel pipe), for a fixed expansion in a direction at right angles to $l_{1}$, which will be called the principal expansion, and, for a zero expansion, in the direction of $l_{1}$ or at right angles to $l_{2}$, which will be called the secondary expansion. If the secondary expansion must be considered, it may be calculated from the equations which have been derived, or one may make use of the curves, Figs. 23 and 24, which are to be used in connection with the curves, Figs. 6 to 22 , as will be explained later. It will be noted that the lengths of pipe, $l_{1}$ and $l_{2}$, are given in feet in all the diagrams, although, in the equations, the units are in inches.

Consider first the expansion in a 4-in. pipe, where the principal expansion is 6 in., and where the secondary expansion (or that at right angles to the principal expansion), is negligible. This length can be determined directly from Fig. 13. If $l_{2}$ is zero, the length of $l_{1}$ will be 37 ft . If $l_{2}$ equals 10 ft ., it will be more than $\frac{l_{1}}{4}$, because $l_{1}$ must equal something less than 37 ft ., if $l_{2}$ has any value at all. With $l_{2}$ equal to 10 ft ., $l_{2}$ is more than $\frac{l_{1}}{4}$ and undoubtedly less than $\frac{l_{1}}{2}$. If $l_{2}$ equals $\frac{l_{1}}{2}$, it will be seen from Fig. 13, that $l_{1}$ will be a little less than 34 ft ., and


Fig. 6.


Fig. 7.


Fig. 8.


Fig. 9.


Fig. 10.


Fig. 11.


Fig. 12.


Fig. 13.


Fig. 14.


Fig. 15.


Fig. 16.


Fig. 17.


Fig. 18.


Fig. 19.


Fig. 20.


Fig. 21.


Fig. 22.


Fig. 23.


Fig. 24.


Fig. 25.
if $l_{2}$ equals $\frac{l_{1}}{4}, l_{1}$ equals 35 ft . It will be seen, therefore, that $l_{2}$ is about two-sevenths of $l_{1}$ or nearly $\frac{l_{1}}{4}$, and that $l_{1}$ should be taken as $34 \frac{1}{2}$ or 35 ft .

If the length, $l_{2}$, becomes equal to $8 l_{1}$, it will be seen from Fig. 13 that $l_{1}$ should equal $27 \frac{1}{2} \mathrm{ft}$. When $l_{2}$ becomes very long, the fact as to whether it can move freely must, of course, be taken into consideration. If it must slip over fixed supports, the friction of slippage will, of course, have the same effect as the shortening of the length, $l_{2}$. On the other hand, it often happens that pipes passing through walls, or in similar conditions, will be held so as to permit of some slight side movement, although it may not be allowed for in the calculations. This, of course, would have the same effect as an increase in the length of pipe under strain.

Let us now consider the expansion in a 4 -in. pipe where the principal expansion is 6 in., the secondary expansion is 3 in ., and the lengths of $l_{1}$ and $l_{2}$ are to be equal. From Fig. $24^{1}$, it will be seen, when the principal expansion divided by the lesser or secondary expansion (or $\frac{s}{r}$ ) $=\frac{1}{2}$, and when $l_{1}=l_{2}$, that (compared with the case where the secondary expansion equals zero) the lengths should be increased in the ratio of 1 to 1.085 , or, from Fig. 23, it will be seen that the expansion should be decreased in the ratio of 1 to 0.85 .

From Fig. 13 it will be seen that $l_{1}$ should be nearly 32 ft . for a 4 -in. pipe with a 6 -in. expansion (the secondary expansion being zero and $l_{1}=l_{2}$ ). If, in addition to the principal expansion of 6 in., there is a secondary expansion of 3 in ., the lengths should be increased, as before stated, in the ratio of 1 to 1.085 , or 32 ft . should be changed to nearly 35 ft . If, however, it is desired not to increase these lengths, the lessened amount of expansion or the increased maximum fiber stress can be determined. From Fig. 23 it has been determined that the expansion should be decreased in the ratio of 1 to 0.85 , or 6 and 3 in. should become 5.1 and 2.55 in ., respectively. If, however, it is desired to maintain the 6 - and 3 -in. expansion, it will be seen from the equations that the fiber stress varies directly as the expansion, and hence the

[^0]maximum fiber stress will become ${ }^{2}$
$$
12000 \times \frac{1}{0.85}=14118 \mathrm{lb} . \text { per sq. in. }
$$

There is one factor which has already been mentioned, to which, however, further attention should be given, that is the question as to how much allowance should be made for the weakening of a pipe at a fitting due either to the strength of the fitting or the threads on the pipe. In order to determine what this allowance should be, we must make $\frac{f I}{\frac{1}{2} D}$, in Equation 22, equal to the moment where $X=k L$, instead of to $M_{1}$, where $X=0$. Following through similar succeeding equations, we will secure the equation:
or

$$
\begin{gather*}
-r=\frac{f}{3 E D} L^{2} k(k-1)-h k  \tag{34}\\
r+s \frac{k}{1-k}=\frac{f l_{1}{ }^{2}}{3 E D k} . \tag{35}
\end{gather*}
$$

The comparative values of the pipe strain at the joint and the fixed point of the pipe, have been determined from Equation 35, and are shown graphically in Fig. 25.

In using this diagram in the following discussion, the loss in strength at the elbow is calculated as one-third. Therefore, an allowance should be made for the weakening at the fitting when the maximum strain is less than $\frac{1}{1-\frac{1}{3}}$, or $1 \frac{1}{2}$ times the strain at the elbow. An examination of the curves on Fig. 25 will show that this is practically always the case when $l_{2}=l_{1}$, or anything less than $l_{1}$. It is also the case when $l_{2}=2 l_{1}$, and the secondary is equal to or larger than the principal expansion. It will be remembered that the principal expansion is usually larger than $\left(\frac{l_{1}}{l_{2}}\right)^{2}$ times the secondary expansion, where $l_{1}$ is the length of pipe under strain at right angles to the direction of the principal expansion, and $l_{2}$ is the length of pipe under strain at right angles to the secondary expansion, but the principal expansion is not necessarily larger in quantity than the secondary expansion.

The allowance will depend on the ratio of the maximum strain to the strain at the elbow, which values are shown by the curves in Fig. 25.
2. Original lacks 'lb.'

When $l_{2}=0$, the strain at the elbow equals the strain at the point where the pipe is held in line, and the strain at all intermediate points is also the same. The allowance for the weakening at the elbow, or for any joint between the elbow and the point where the pipe is held in line, therefore, should be in the ratio of 1 to $1 \frac{1}{2}$ or the strain allowable should be two-thirds of what otherwise might be calculated when $l_{2}=0$.

If $l_{2}$ has some other value, for example, if $l_{2}=2 l_{1}$, and if $\frac{s}{r}=2$, it will be seen, from Fig. 25, that the maximum strain is $1 \frac{1}{4}$ times the strain at the elbow. If we calculate the elbow to be only two-thirds as strong as the pipe, the strain in the latter should be reduced to $\frac{2}{3} \times 1 \frac{1}{4}$ $\left(=\frac{5}{6}\right)$ of what the pipe might stand, so as not to overstrain the material at the elbow. If there is a fitting half way between the elbow and the point at which the pipe is held in line, the maximum strain would be greater than the strain at this point, in the ratio of 1 to a quantity half way between 1 and $1 \frac{1}{4}$, or in the ratio of 1 to $1 \frac{1}{8}$.

If the fitting were farther from the elbow, its distance being twothirds of the total distance between the point at which the pipe is held in line and the elbow, the ratio would be 1 to $\left(1+\frac{1}{3}\right) \frac{1}{4}$, which is equal to the ratio of 1 to $1 \frac{1}{12}$.

When $l_{2}=0$, therefore, the allowance for the weakening at the elbow should be made as follows:

$$
\begin{aligned}
& \frac{\text { The new } l_{1}{ }^{2} \text { and } l_{2}{ }^{2}}{\text { The old } l_{1}{ }^{2} \text { and } l_{2}{ }^{2}}=\frac{1}{\frac{2}{3}} \text {, or, } \\
& \frac{\text { The new } l_{1} \text { and } l_{2}}{\text { The old } l_{1} \text { and } l_{2}}=\sqrt{\frac{3}{2}}=\sqrt{1.5}=1.225,
\end{aligned}
$$

or the new lengths should equal 1.225 times the old lengths.
An examination of the equation and curves will also show that this same rule may be followed closely when the secondary expansion, although not zero, is still small as compared with $\left(\frac{l_{2}}{l_{1}}\right)^{2}$ times the principal expansion, and, in no case, will the allowance be greater than this amount.

The diagrams may be used for a variety of combinations, and their adaptability is well illustrated by Fig. 26.

An arrangement of piping is assumed, in which a tee connects an 8 - and a 6 -in. pipe on the run and a 4 -in. pipe branches from the
tee at right angles to both. The length of the 8 -in. pipe from the tee to the point at which it is held in line will be assumed to be 30 ft ., and that of the 6 -in. pipe will be assumed to be 25 ft . If the movement of the tee in the direction of the 6 - and 8in. pipes is 2 in., what should the length of the 4 -in. pipe be from the tee to the point at which it is held in line?

It will first be assumed that the expansion of the tee in the direction of the 4 -in. pipe is a negligible quantity. From the formula it is seen that the bending


Fig. 26. for a fixed maximum fiber stress varies inversely as the diameter of the pipe and the square of the length of pipe. The 30 ft . of 8-in. pipe, therefore, could, for a fixed maximum fiber stress, be bent only one-half as much as 30 ft . of 4 -in. pipe. Therefore, the 30 ft . of 8-in. pipe is equivalent in its power to bend to 21.2 ft . of 4 -in. pipe, since the lengths vary as the square root of the bending power, and $\frac{21.2}{30}=\sqrt{\frac{1}{2}}$. In the same way, the 25 ft . of 6 -in. pipe is equivalent to $\frac{25}{\sqrt{1.5}}=\frac{25}{1.224}=20 \mathrm{ft}$. (about).

We have now the double stiffness of an equivalent of 21.2 ft . of 4 -in. pipe and 20 ft . of 4 -in. pipe. The stiffness is proportional to $\frac{1}{l^{2}}$, or to the reciprocal of the length squared. Therefore, the combined stiffness of the two 6 - and 8 -in. pipes would be equivalent to $\frac{1}{(21.2)^{2}}+\frac{1}{20^{2}}=$ $\frac{849.44}{179776}$, which would be equivalent to a 4-in. pipe, with a length of $\sqrt{\frac{179776}{849.44}}=\sqrt{211.64}=14.54 \mathrm{ft}$.

If we use 14.54 as $l_{2}$, therefore, in Fig. 13 , we can secure the necessary length of 4-in. pipe.

We see from this diagram that, if $l_{2}=0,2 \mathrm{in}$. of expansion for a 4 -in. pipe would require a length of $21 \frac{1}{4} \mathrm{ft}$. of 4 -in. pipe. If $l_{2}=l_{1}, l_{1}$ would be $18 \frac{1}{8} \mathrm{ft}$., but $l_{2}$ is equivalent to 14.5 ft . The next smaller value for $l_{2}$ is $l_{2}$ equals $\frac{1}{2} l_{1}$. In this case, $l_{1}$ would equal $19 \frac{3}{8} \mathrm{ft}$. In the first
case, if $l_{2}=l_{1}, l_{2}$ would be $18 \frac{1}{8}$. In the second case, if $l_{2}=\frac{1}{2} l_{1}, l_{2}$ would be $9 \frac{11}{16} \mathrm{ft}$., but $l_{2}$ is actually $14 \frac{1}{2} \mathrm{ft}$., or about half-way between the two; therefore, $l_{1}$ lies between $19 \frac{3}{8}$ and $18 \frac{1}{8}$, and would not be far from 19 ft .

If now there should be a secondary expansion of $\frac{1}{2}$ in., it would be necessary to increase the value of $l_{1}$. If $l_{2}$ could not be increased proportionately, the value of $\frac{l_{2}}{l_{1}}$ would be decreased, and instead of being $\frac{14 \frac{1}{2}}{19}$, or approximately $\frac{3}{4}$, it would become something less. From Fig. 24 it will be seen that when the secondary expansion divided by the principal expansion, or $\frac{s}{r}=\frac{1}{4}$, and when $l_{2}=\frac{1}{2} l_{1}$, that the new length should be about 1.1 of the old length. This, however, would change $l_{1}$ so little that the value of $\frac{l_{2}}{l_{1}}$, while less than $\frac{3}{4}$, would not be nearly so small as $\frac{1}{2}$. We can, therefore, approximate the value of $\frac{l_{2}}{l_{1}}$ as $\frac{9}{10} \times \frac{3}{4}=\frac{27}{40}=0.675$, or about $\frac{2}{3}$, and we can approximate the required length of $l_{1}$ from Fig. 24, as about 1.09 of the old length of 19 ft . or the new length of $l_{1}$ would become about $19 \times 1.09=20.71$, or about $20 \frac{3}{4} \mathrm{ft}$.

Let us now consider a case in which the strain is not caused by an expansion or movement at the tee, and in which the end of the $4-\mathrm{in}$. pipe farthest from the tee is not held in line. The problem will be to find what bending is possible at the end of the 4 -in. pipe farthest from the tee, if this end is not held horizontal or in its original direction. The length of the 8 - and 6 -in. pipes will be the same as before, and the length of the 4 -in. pipe will be the same as in the last case, viz., $20 \frac{3}{4} \mathrm{ft}$.

In this case the 8 - and 6 -in. pipes together have an equivalent length of 14.54 ft . of $4-\mathrm{in}$. pipe.

It is apparent that, in this case, the bending will be the same as for a 4 -in. pipe of length $(20.7+14.54)=35.24 \mathrm{ft}$. with its end free. A free end is a similar condition to $l_{2}=\infty$, as the fact of the ends not being free is due to a strain from $l_{2}$, and this becomes zero when $l_{2}=\infty$. We will find, therefore, the expansion on Fig. 13, and note that it is nearly 11 in.

It is sometimes the practice to use bends. An analysis of the conditions of strain in a $90^{\circ}$ bend will show that such a bend has approximately the same strength as a pipe making a $90^{\circ}$ turn with an elbow,
the two sides of which are each equal to the radius of the bend. There are advantages, however, in the use of the bend-a lessening in the resistance to the flow of the fluid in the pipe, a lessening in the number of fittings used, and, in many cases, a lessening in the cost of the material and erection. In some specifications copper bends are called for, and if the pipe is under a practically constant temperature, these may be fairly satisfactory, even if the stress in the metal is quite large. If the stress is kept fairly well under the elastic limit, steel pipe is, however, as good as copper pipe, and if the elastic limit is nearly reached or exceeded, there is always danger of the breaking of the pipe.

If the pipe is arranged as shown in Fig. 27, and if the expansion is the same on both sides of the loop, the center of the loop will act as a fixed point, and $l_{1}$ will be the length of one side of the loop. If now the expansion were altogether on one side of the loop, the entire length of both sides may be taken for $l_{1}$, which would appear to make it advantageous to place the loop near an anchorage instead of half way between two anchorages. The reason for this is that by placing the loop near one anchorage, we double the expansion acting on one side of the loop, but we


Fig. 27. also double $l_{1}$, and the expansion permissible varies as $l_{1}^{2}$ and not as $l_{1}$. If, however, the loop is placed half way between the two anchorages, the greatest movement of the pipe at any point is reduced by one-half, and, at times, this may be of considerable importance.

The method of using several small pipes or bends in place of one large pipe or bend has been suggested, and can sometimes be used to advantage. The intention is, of course, to make this section of piping capable of greater bending, and, at the same time, not to reduce the area through which the steam or fluid passes. The combined crosssectional area of the small pipes may be made equal or larger than that of the large pipe, while the bending will be equal to that of one small pipe.

There is one point in connection with the expansion of pipes, to which particular attention is called. It is possible to reduce greatly the effects of expansion by the use of what is called a cold strain. William J. Baldwin, M. Am. Soc. C. E., was the first, the writer believes, to
make use of cold strain in order to reduce the necessary allowance for expansion, and he has gone as far as to put the entire strain in the cold pipe in some cases.

By cold strain is meant the cutting of the pipe in such a manner that the pipe will be strained when cold in exactly the opposite way from that in which it is strained by expansion when hot. If the cold strain is $50 \%$ of the normal expansion, the pipe will be strained $50 \%$ in what may be called a negative direction when cold, and $50 \%$ in the opposite or positive direction when hot. By this means it is possible to reduce the strain in the pipe to half the normal strain of expansion, and thus reduce the necessary allowance for the same in this proportion. There are advantages, however, in making the cold strain greater than 50 per cent. If, for instance, the cold strain is exactly equal to the expansion, then the pipe is under strain when cold and entirely free from strain when hot or, in other words, the tendency to strain open one side of a pipe flange is a maximum when the pipe is cold or when there is no pressure on it, and it becomes zero when the pipe is hot or when there is a maximum pressure on it. The strain of expansion is also eliminated when that due to steam pressure is a maximum, so that the pipe is not subjected to the two strains at the same time.

If the cold strain is something less than the full expansion, these effects are, of course, proportionately decreased.

Perhaps it might be of interest to mention one instance in which the advantage of cold strain was used to remedy what seemed to be a serious trouble. A leak in one of the main steam pipes in a large hotel was causing much annoyance. The cause of the trouble was simply that the flanges were thrown out of line by the strain of the expansion of the piping. The location was such that it was of the utmost importance not to shut off the steam from this pipe for more than a very short time. Various expedients had been suggested, all requiring a shut-down of the plant for a considerable length of time, when it was suggested by Mr. Baldwin, that one length of flanged pipe be replaced by a similar piece of pipe, cut enough shorter, however, to eliminate entirely the excessive strain of expansion when the piping was hot. This new piece of pipe was made ready before the plant was shut down, and the time, therefore, during which it was not in operation, was limited to a very few minutes.

There is another advantage due to cold strain, which may be men-
tioned, and which will be appreciated particularly by the steamfitter or the man in charge of the erection of the work, wherever flanged piping is used. The advantage is simply this, that a flange joint when unbolted will tend to open up and allow an easy removal of a gasket.

In conclusion, a word may be said in regard to the use of the diagrams. They are calculated for a maximum fiber stress of 12000 lb . per sq. in. This gives an ample factor of safety for wrought-iron or steel pipe, and more, perhaps, than some will wish to allow. It is very easy, however, to use the diagrams with a higher stress, if so desired, since the stress varies directly as the amount of expansion. If it is desired, for instance, to use a maximum fiber stress of 16000 lb . per sq. in., it is only necessary to increase the amounts of expansion in the diagrams in the ratio of 16000 to 12000 , or $\frac{4}{3}$. An expansion of 3 in . in the diagrams can then be made 4 in ., and other values can be increased proportionately.

## DISCUSSION.

Mr. Ennis.
William D. Ennis, Esq.* (by letter).-Mr. Taggart has rendered a real service in reducing to quantitative form the various assumptions which engineers have had to make in designing pipe lines in order to provide for expansion. There is no question as to the benefit, in some cases, of merely treating a long transmission line as suggested in Fig. 27; but this seems to be a clumsy expedient at best, to be resorted to only as a means of getting around a difficulty quickly and cheaply. The use of cold strain in erection has been thoroughly tried out by the fitters, and always with success; it cannot be too strongly insisted on that all piping should be erected in this way.

The writer does not follow Mr. Taggart's apparent condemnation of the bend as an expedient for expansion resistance, having supposed that its shape, ordinarily at least, gives it a susceptibility to flexure greater than that of the straight pipe with elbows. Certainly there is less likelihood of leakage at the end joints when bends are used. If this is not due to greater flexibility, on what grounds is it to be explained?

Both cold straining and expansion strains have an important relation to flange pressure. The ordinary pipe flange, with a continuous face, has a contact pressure due to bolting only slightly in excess of that necessary to hold it against high strain pressures. Unless the question of anchorage is carefully worked out, cases sometimes arise in which the cold strain or the expansion may compel a flange to leak.
mr. Kent. William Kent, Esq. (by letter).-It seems to the writer that the theory of the action of the pipes shown in Fig. 2 is not correct. Why should the pipe, $b d$, take the reverse curve there shown? If both pipes expand, and the elbow, $a$, moves to $b$, they would probably curve in a single direction, as shown in Fig. 28, tending to bend the right-angled elbow into an acute angle, or, if


Fig. 28. that is too stiff, to open the joints

[^1]of the flanges if they are separated by gaskets, or to crack the flanges if they are metal to metal. If screwed elbows are used, the screwed ends of the pipes, being their weakest part, would tend to be distorted.

All the sketches seem to relate to practice which is no longer considered good, except for small pipes and low pressures. Fifteen or twenty years ago the expansion joint shown in Fig. 27 was not uncommon, but many accidents resulted from it, not only from the cracking of the flanges due to expansion, but also from water-hammer. When steam pressures began to be 120 lb . and upward, and high-speed engines became common, accidents to steam pipes became most serious and frequent, and led to the adoption of long steel bends and forged-steel flanges, with no cast-iron or screwed joints. Steam-pipe lines, formerly the most dangerous part of a steam plant (accidents to them being more numerous than boiler explosions or bursting fly-wheels), have now become most reliable, an accident to a well-constructed modern pipe line being very rare.

What is needed for the aid of designers is a series of experiments on long pipe bends like those in Fig. 29, in order to ascertain how much expansion will be safely taken care of by their flexibility.

Referring to Mr. Taggart's criticism of taking care of expansion by swinging fittings, such fittings have sometimes been used with excellent results. A


Fig. 29. noted instance is that of the steam-pipe line of the Waldorf-Astoria Hotel, in which the pipe expansion was taken care of by right-angled bends and by loops like that shown in Fig. 27, the result being scores of cracked flanges, leaky screwed joints, and the like. The whole piping system was condemned as highly dangerous by several engineers who examined it, and it was relaid with swinging fittings, a few years after the hotel was built, with complete success, the writer has been informed. Expansion joints which have two pipe legs, connected with a sort of ball joint, are now on the market, and are giving good service, even with superheated steam at high pressure.

Ralph C. Taggart, Assoc. M. Am. Soc. C. E. (by letter).- mr. Taggart. Mr. Ennis and Mr. Kent have both drawn one conclusion from the paper which certainly was not intended, namely, that the writer is opposed to the use of pipe bends. The writer is not opposed to pipe bends; in fact, he is a strong advocate of their use in many cases. In the paper,

Mr. Taggart. however, he desired to make clear the fact that pipe bends do not add to the flexibility of piping because of their shape. They often add to the possible flexibility of the piping, however, by the elimination of fittings, where the latter may be a source of weakness. This question was dealt with at some length in connection with Fig. 25, and, for this reason, the writer did not go into the question of the weakness of fittings when discussing pipe bends.

Any arrangement which will remove a fitting from a point of maximum strain is desirable. The location of these points was discussed in connection with Fig. 25. Such points, however, are not always located where the change of direction occurs.

The writer uses the right-angle diagrams because of their simplicity, and because the curves derived therefrom may be used, whether elbows or bends are installed.

A careful analysis of the stresses and strains in a bent pipe, considered as an elastic arch, will show that the radius of the bend required for a fixed maximum fiber stress is practically the same as the length of one side of a square corner, where the turn in the pipe is as strong as the pipe itself. If the ends of the bends are held rigidly, both as to alignment and position, the analysis will show that the bend with a radius, $r$, will be more rigid than the square corner of a length, $l$, on each side, where $l$ equals $r$. This is an unusual condition, however, and the bend may practically be calculated as if the pipe ran to a square corner.

In regard to experiments on pipe bends which have been made in the past, attention is called to one fact, namely, that, where these tests are carried to a point at which the pipe ruptures, the pipe usually swings out of its original plane and bends in at least two planes. This results in greater bending in the pipe, before it breaks, than is shown by the calculations made on the basis of bending in a single plane; but when it is desired to keep the maximum fiber stress in the pipe down to what is generally considered to be good practice in steel structures, practically all the expansion occurs within one plane, and this is often essential on account of the alignment of the piping. The writer wishes to bring out the fact that, if the stresses are calculated from experiment, based on the point at or near which the pipe ruptures (bending in two planes), the stresses for lesser expansions, figured therefrom, will be too high, where now the pipe bends only within one plane. With this fact in
mind, the writer believes that experiments on the expansion of pipes Mr. Taggart. confirm the data in the paper.

Mr. Kent states that the theory of the action of the pipes under expansion shown in Fig. 2 does not seem to him to be correct. He asks: "Why should the pipe, $b d$, take the reverse curve there shown?" He suggests that, with expansion in two directions, the pipe would probably take the position shown in Fig. 28.

The reason for the pipes taking the reverse curve, and for the condition shown in Fig. 28 being incorrect, is simply that the flanges must not be allowed to open as shown in that figure. It is ordinarily an impossible condition, if the pipe is to carry steam. The flanges must be made to come together, and this tightening-up of the bolts on the outer sides of the flanges is what must necessarily put the reverse curve in the pipe.

If in any special case the condition cited by Mr. Kent occurs, the resulting strains may still be found from the diagrams given. The condition is simply one in which the bending moment at the fitting becomes zero. This is the same as the condition in which $l_{2}$ equals $\infty$, and is shown by the curves which are thus marked.

The condition shown in Fig. 2 is for an expansion in one direction only, in which case the maximum strain does not come at the elbow. That shown in Fig. 28 is for an equal expansion in two directions, in which case the maximum strain comes at the elbow. The actual position of the pipe under the conditions represented in Fig. 28 would be that shown in Fig. 30.

Mr. Kent seems to think that the writer's sketches do not refer to and cannot be used in connection with bends. In this he is mistaken. The curves which Mr. Kent shows in Fig. 29 can be calculated by the diagrams in the paper. The writer would determine their flexibility by calculating for a straightened pipe with $90^{\circ}$ turns from each end of the bend to a point $90^{\circ}$ from the direction of expansion. The remainder of the pipe is calculated directly on the basis of its length. For example, the bends shown in Fig. 29 are shown in Fig. 31.

The dotted lines from $a$ to $b$ and from $d$ to $e$ show how the pipe should be figured up to the points $b$ and $d$, if the expansion is in the direction of the pipe at the end of the bends, or as shown by the arrows. The pipe between $b-c-d$ can be calculated directly on the basis of its length, and as if it were at right angles to the direction of the expansion.


Fig. 30.


Fig. 31.

Mr. Taggart. If the expansion is at right angles to the direction of the arrows, the dotted lines would not be considered, and the entire length of the curved pipe would, of course, be calculated as if it were at right angles to the expansion.

Mr. Kent seems to think that the writer's sketches refer only to low-pressure work. This is not true. As before stated, they may be applied to bends and to piping as arranged for the highest pressures.

A great deal of piping is installed to-day with pressures of more than 120 lb . per sq. in., where elbows are used, which give satisfactory service. For the higher pressures, cast-steel, and not cast-iron, elbows are used.

The lessening, in recent years, in accidents from steam piping under high pressure, has been due largely to the more general use of steel in place of cast iron. Cast iron should never be used in piping under very high pressure because of its uncertainty. Water hammer will also injure cast iron much sooner than steel on account of its brittleness. Water hammer in steam piping, however, is a crime. Steel pipe and bends in high-pressure piping are of advantage, but if a serious water hammer occurs it is almost sure to cause trouble.

In regard to expansion allowances, it may be said that the designer who has had experience may turn out a very satisfactory equipment, but in too many cases the factor of safety is too low. It is almost beyond the range of human possibility for experience alone to maintain a uniform factor of safety, and exact calculations should always be made where there is the slightest doubt as to the amount of allowance for expansion.

In regard to the use of swinging fittings where allowance must be Mr. Taggart. made for pipe expansion, the writer does not agree with Mr. Kent. He does not see how any unprejudiced person can reach a different conclusion from that contained in his paper, and knows that practically all operating engineers and steam-fitters of experience will condemn such joints. The only argument that he has ever heard in their favor is the statement that they have been used somewhere with success. Mr. Kent cites one instance. The writer went to the building mentioned, where the engineer described the old arrangement of the piping. It appears that the original installation contained standard-weight pipe, although the plant itself is a high-pressure steam plant. There was trouble, not only at the expansion loop, but at practically every other joint in the piping. The expansion loop itself was entirely too small to do any appreciable good. New high-pressure piping, all extra heavy, was installed. In the new arrangement the piping was run so as to allow for a swinging joint. Whether there is a movement at this joint, however, is unknown, as a constant steam pressure is maintained practically at all times. The engineer told the writer that he did not believe there was any movement within the thread of the fitting, and the writer's own observation leads him to believe that statement, the expansion being carried largely to the ends of the piping.

If, however, there is in any single case or in any number of cases a movement in the thread of the fitting without leakage, this certainly is the exception and not the rule. In the majority of cases, where there is no trouble with what are thought to be swinging joints it will be found that there is no movement in the thread, although such may be imagined.

In regard to the use of ball-and-socket joints as swinging expansion joints, the writer will only quote from a statement made by a company which has claimed in the past that it is the only successful all-metal flexible ball-joint manufacturer in the United States. The company states, "Will guarantee the joint if used constantly, unless you move the joint exactly in the same place for a long time." This condition appears to be the one that must be met in most cases where such a joint would be used as an expansion joint.

In connection with pipe expansion, there is a method of shortening expansion loops which is not mentioned in the paper. It consists in the use of standard-weight pipe for the bends or expansion loops, while the

Mr. Taggart. remainder of the piping is extra heavy. An extra heavy pipe will bend as many inches as a lighter pipe of the same outside diameter, but it takes more force to produce this bending. The use of the lighter pipe bends, therefore, throws less strain on the other piping. It is much better to use longer loops, however, and maintain the extra heavy piping throughout.

In conclusion, a word should be added in regard to what were termed primary and secondary expansions. An approximate method of determining the primary or principal expansion was given in the paper. Usually, this is only a matter of simple observation; but, when there is any question as to which expansion is the primary and which is the secondary, it is a simple matter to calculate the piping both ways and take the values which show the greater strains.

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[^0]:    1. The curve for $l_{2}=4 l_{1}$ in Fig. 24 has been corrected. The original figure showed the curve passing through $(2,1.05)$
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